Sri Lanka Institute of Information Technology

IT0060 –Essential Mathematics

Ms. Pubudika Wijesundara
Ms. Shashini Kumarasinghe

Trigonometry

Outline

- Trigonometry Angles
- Arc Length
- Area of a Sector of a Circle
- Right Triangle Trigonometry
- Trigonometric ratios
- Fundamental Trigonometric Identities
- Trigonometric identities

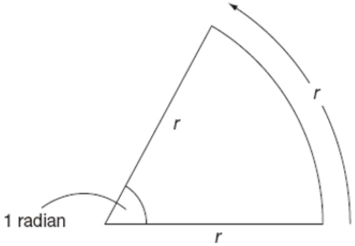
Trigonometry Angles

Angles can be measured in two measurements:

- Degrees
- Radians

Relationship between Degrees and Radians

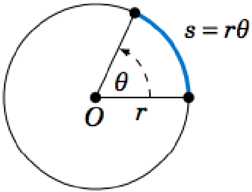
$$360^0 = 2\pi\, rad$$



Arc Length

The arc length of a circle (s) is a portion of its circumference, calculated by multiplying the radius (r) by the central angle (θ). It represents the linear distance along the curved edge.

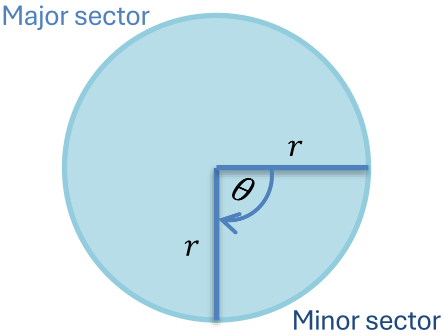
Arc Length Formula (if θ is in radians)	$s = r\theta$
Arc Length Formula (if θ is in degrees)	$s = 2\pi r(\theta/360^o)$



Area of a Sector of a Circle

The sector is basically a portion of a circle. It divides the circle into two regions, namely Major (more than 180^o) and Minor (less than 180^o) Sector.

$$Area = \frac{\theta}{360^o}\pi r^2$$

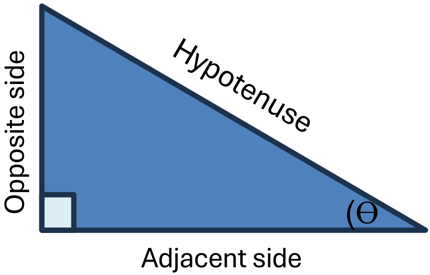


Right Triangle Trigonometry

A right-angled triangle is a triangle with one internal angle measuring exactly 90 degrees (a right angle), identified by a small square symbol.

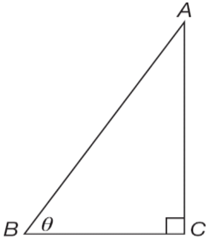
Relative to the angle θ the three sides of the right-angled triangle are,

- Hypotenuse
- Opposite side
- Adjacent side



Trigonometric Ratios

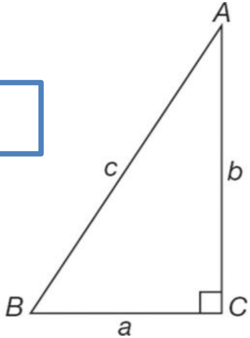
Abbreviation	Relationship to sides of a triangle
sin	AC/AB
cos	BC/AC
tan	AC/BC
cosec	AB/AC [1/sin(θ)]
sec	AB/BC [1/cos(θ)]
cot	BC/AC [1/tan(θ)]



Fundamental Trigonometric Identity: Pythagoras Theorem

Pythagoras theorem states that "**In a right-angled triangle, the square of the hypotenuse side is equal to the sum of squares of the other two sides**".

$$a^2 + b^2 = c^2$$



Fundamental Trigonometric Identity

Trigonometric Identities are true for every value of variables occurring on both sides of an equation.

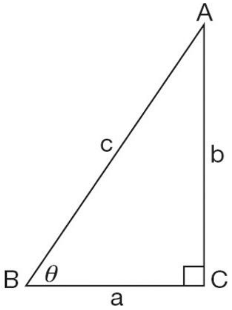
The fundamental trigonometric identity, often called the **Pythagorean Identity,** states the relationship between sine and cosine, forming the basis for simplifying expressions and solving equations.

$$a^2 + b^2 = c^2$$

$$\frac{a^2}{c^2} + \frac{b^2}{c^2} = 1$$

$$cos^2\theta + sin^2\theta = 1$$

$$\boxed{sin^2\theta + cos^2\theta = 1}$$



Trigonometric Identities

Dividing the fundamental identity by $\cos^2\theta$

$$cos^2\theta + sin^2\theta = 1$$

$$\frac{cos^2\theta}{cos^2\theta} + \frac{sin^2\theta}{cos^2\theta} = \frac{1}{cos^2\theta}$$

$$\boxed{1 + tan^2\theta = sec^2\theta}$$

Trigonometric Identities

Dividing the fundamental identity by $\sin^2\theta$

$$cos^2\theta + sin^2\theta = 1$$

$$\frac{cos^2\theta}{sin^2\theta} + \frac{sin^2\theta}{sin^2\theta} = \frac{1}{sin^2\theta}$$

$$\boxed{cosec^2\theta + 1 = cot^2\theta}$$

Trigonometric Identities

Sums and differences of angles

$$\sin(\theta + \emptyset) = sin\theta cos\emptyset + cos\theta sin\emptyset$$
$$\sin(\theta - \emptyset) = sin\theta cos\emptyset - cos\theta sin\emptyset$$

$$\cos(\theta + \emptyset) = cos\theta cos\emptyset - sin\theta sin\emptyset$$
$$\cos(\theta - \emptyset) = cos\theta cos\emptyset + sin\theta sin\emptyset$$

$$\tan(\theta + \emptyset) = \frac{tan\theta + tan\emptyset}{1 - tan\theta tan\emptyset}$$
$$\tan(\theta - \emptyset) = \frac{tan\theta - tan\emptyset}{1 + tan\theta tan\emptyset}$$

Double angles

$$sin2\theta = 2sin\theta cos\theta$$

$$cos2\theta = cos^2\theta - sin^2\theta$$

$$cos2\theta = 1 - 2sin^2\theta$$

$$cos2\theta = 2cos^2\theta - 1$$

$$tan2\theta = \frac{2tan\theta}{1 - tan^2\theta}$$

Use of Trigonometric Identities Example

Prove that $tan\, x + cot\, x = \dfrac{1}{sin\, x.cos\, x}$

$$
\begin{aligned}
LHS &= \tan x + \cot x \\
&= \frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} \\
&= \frac{\sin x . \sin x + \cos x . cos x}{\sin x . \cos x} \\
&= \frac{sin^2 x + cos^2 x}{\sin x . \cos x} \\
&= \frac{1}{\sin x . \cos x} \\
&= RHS
\end{aligned}
$$

Use of Trigonometric Identities Example

Prove that $sin^2x = \frac{1-cos2x}{2}$

$$
\begin{aligned}
RHS &= \frac{1-cos2x}{2} \\
&= \frac{1-\left(1-2sin^2x\right)}{2} \\
&= \frac{2sin^2x}{2} \\
&= sin^2x \\
&= LHS
\end{aligned}
$$

Thank You!